ON GALKIN'S LOWER BOUND CONJECTURE

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ABSTRACT. We estimate an upper bound of the spectral radius of a linear operator on the quantum cohomology of the toric Fano manifolds $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O} \oplus \mathcal{O}(3))$. This provides a negative answer to Galkin's lower bound conjecture.

1. INTRODUCTION

The first Chern class of a Fano manifold X induces a linear operator $\hat{c}_1$ on the even part $H^{\mathrm{ev}}(X)$ of the classical cohomology ring $H^*(X, \mathbb{C})$ by

$$\hat{c}_1 : H^{\mathrm{ev}}(X) \longrightarrow H^{\mathrm{ev}}(X); \alpha \mapsto (c_1(X) \star \alpha)|_{\mathbf{q}=1}.$$

Here $\star$ denotes the quantum multiplication, which involves genus-zero, three-pointed Gromov-Witten invariants of X, and $\mathbf{q}$ denote the quantum variables. It is important to study the distribution of eigenvalues of $\hat{c}_1$. Indeed in [GGI], Galkin, Golyshev and Iritani proposed remarkable Conjecture $\mathcal{O}$ and the relevant Gamma conjecture I and II. Conjecture $\mathcal{O}$ concerns about the spectral radius

$$\rho = \rho(\hat{c}_1) := \max\{|\lambda| \mid \lambda \text{ is an eigenvalue of } \hat{c}_1\}.$$

There is another relevant lower bound conjecture proposed by Galkin [Ga1].

Galkin's Lower Bound Conjecture. *For any Fano manifold X, we have*

$$\rho \geq \dim X + 1 \textit{ with equality if and only if } X \cong \mathbb{P}^n.$$

This conjecture has been verified for some cases [ESSSW, ChHa, ShWa, Ke, HKLS], and can also be supported by the numerical computations based on the analysis in [Yang] for the blowup of $\mathbb{P}^n$ along $\mathbb{P}^r$.

As the main result of this paper, we provide a negative answer to the above conjecture, by showing the following for the toric Fano manifolds $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O} \oplus \mathcal{O}(3))$.

Theorem 1.1. *Let $X = \mathbb{P}_{\mathbb{P}^n}(\mathcal{O} \oplus \mathcal{O}(3))$, where n is sufficiently large with $3 \nmid n+1$. Then $\rho < \dim X + 1 = n + 2$.*

We will give a more precise statement in **Theorem 2.8**. To prove the theorem, we use mirror symmetry for toric Fano manifolds, especially the fact [Au, OsTy] that the eigenvalues of $\hat{c}_1$ are in one-to-one correspondence with the critical values of the Hori-Vafa superpotential [HoVa] mirror to the toric Fano manifold X. We further reduce the spectral radius to an optimization problem in nonlinear programming, and then achieve the aim by classical analysis. This approach was used in [GHIKLS], leading to negative answers to Conjecture $\mathcal{O}$ and Gamma conjecture I therein.

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2. Galkin's lower bound conjecture for $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(3))$

This section is devoted to finding an upper bound of the spectral radius for the toric Fano manifolds $X=\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(3))$ with n sufficiently large, as we will see in Theorem 2.8.

2.1. **Mirror symmetry for X.** Note that X is a $\mathbb{P}^1$-bundle over $\mathbb{P}^n$, which is of dimension $n+1$ and of Picard number two. The classical cohomology ring $H^*(X)=H^*(X,\mathbb{C})$ is of dimension $2n+2$, and $H^*(X)=H^{\mathrm{ev}}(X)$ does not contain nonzero classes of odd degree.

Moreover, X is a toric Fano manifold, whose associated fan in $\mathbb{R}^{n+1}$ has exactly $(n+3)$ primitive ray generators b_i, given by (see e.g. [CLS] for more details on toric geometry)

$$b_i=e_i:=(0,\dots,0,1,0,\dots,0),\quad \text{for } 1\le i\le n+1,$$

$$b_{n+2}=-\sum_{i=1}^{n}e_i+3e_{n+1},\qquad b_{n+3}=-e_{n+1}. \tag{1}$$

Consequently, the Hori-Vafa superpotential f mirror to X can be immediately read off, which is a holomorphic function $f:(\mathbb{C}^\times)^{n+1}\to\mathbb{C}$ defined by

$$f(\mathbf{x})=\sum_{i=1}^{n+3}\mathbf{x}^{b_i}=x_1+x_2+\cdots+x_{n+1}+\frac{x_{n+1}^3}{x_1x_2\cdots x_n}+\frac{1}{x_{n+1}}. \tag{2}$$

One remarkable statement in mirror symmetry for X gives a ring isomorphism between $QH^*(X)$ and the Jacobi ring Jac(f). Here $QH^*(X)=(H^*(X)\otimes_{\mathbb{C}}\mathbb{C}[q_1,q_2],\star)$ denotes the (small) quantum cohomology ring of X. It is a deformation of the classical cohomology ring $H^*(X)$, by incorporating genus-zero, three-pointed Gromov-Witten invariants of X into the quantum product (see e.g. [CoKa] for more details). Precisely, there is an isomorphism Ψ of $\mathbb{C}$-algebras [Baty],

$$\Psi: QH^*(X)|_{(q_1,q_2)=(1,1)}\longrightarrow \mathrm{Jac}(f)=\mathbb{C}[x_1^{\pm1},...,x_{n+1}^{\pm1}]/(x_1\partial_{x_1}f,...,x_{n+1}\partial_{x_{n+1}}f).$$

Here we specify $\mathbf{q}=\mathbf{1}$ for only introducing f without deformation. Moreover, we have

Proposition 2.1 ([OsTy, Corollary G]). $\Psi(c_1(X))=[f]$. *Namely, eigenvalues of $\hat{c}_1$ on $QH^*(X)|_{\mathbf{q}=\mathbf{1}}$, with multiplicities counted, coincide with the critical values of f.*

2.2. **Distribution of critical values of f.** Assume 3 and $n+1$ to be coprime, i.e., $3\nmid n+1$.

Critical points $\mathbf{x}=(x_1,\dots,x_{n+1})$ of f are the solutions to the system of equations $\partial_{x_i}f=0$, $1\le i\le n+1$. A simple calculation shows that

$$x_1=x_2=\cdots=x_n=:x\in\mathbb{C}^\times.$$

Denote $y:=x_{n+1}\in\mathbb{C}^\times$. Then the system of equations $\frac{\partial f}{\partial x_i}=0$ can be reduced to

$$1-\frac{y^3}{x^{n+1}}=0,\quad 1+\frac{3y^2}{x^n}-\frac{1}{y^2}=0,\quad (x,y)\in(\mathbb{C}^\times)^2. \tag{3}$$

Since 3 and $n+1$ are coprime, $l_1\cdot 3+l_2\cdot(n+1)=1$ for some integers l_1,l_2. By setting $t=y^{l_2}x^{l_1}$, we obtain the one-to-one parameterization $(x,y)=(t^3,t^{n+1})$ with $t\in\mathbb{C}^\times$

for solutions to the first equation in (3). Therefore the system (3) is equivalent to $h(t) = 1$ with

$$h(t) := t^{2n+2} + 3t^{n+4}. \tag{4}$$

Consequently, the critical values of f (with multiplicity counted) are precisely given by $g(\alpha)$ at the roots α of $h - 1$, and we further notice $g(\alpha) = \tilde{g}(\alpha) = \breve{g}(\alpha)$, where

$$g(t) := nt^3 + t^{n+1} + t^3 + \frac{1}{t^{n+1}}, \quad \tilde{g}(t) := (n-2)t^3 + \frac{2}{t^{n+1}}, \quad \breve{g}(t) := 2t^{n+1} + (n+4)t^3.$$

Therefore we are led to the following optimization problem in nonlinear programming.

Problem 2.2. Maximize $|g(t)|$ subject to $h(t) = 1,\ t \in \mathbb{C}^\times$.

Viewed as a real function, $h(t)$ is strictly increasing on $\mathbb{R}_{\geq 0}$. Note $h(0) < 1 < h(1)$. Hence $h(t) - 1$ has a unique positive single root a_+ with

$$a_+ \in (0, 1).$$

Moreover, the following lemma follows immediately from the observation

$$\lim_{n\to+\infty} h(1 - \frac{1}{n}) = \lim_{n\to+\infty} \big((1 - \frac{1}{n})^{2n+2} + 3(1 - \frac{1}{n})^{n+4}\big) = \frac{1}{e^2} + \frac{3}{e} > 1.$$

Lemma 2.3. *There exists an integer $N_1 > 0$, such that $a_+ < 1 - \frac{1}{n}$ for any $n \geq N_1$.*

Recall Rouché's theorem in classical complex analysis (see, e.g., [GKR, Theorem 6.24]).

Lemma 2.4 (Rouché's theorem)**.** *Let f_1 and f_2 be holomorphic functions on $\{z \in \mathbb{C} \mid |z| \leq R\}$ where $R > 0$. If $|f_2| < |f_1|$ on $\{z \in \mathbb{C} \mid |z| = R\}$, then f_1 and $f_1 + f_2$ have the same number of zeros (counted with multiplicity) in $\{z \in \mathbb{C} \mid |z| \leq R\}$.*

We say that the subset $\{z \in \mathbb{C} \mid |z| \leq 1\}$ is the unit disc.

Proposition 2.5. *There are exactly $n + 4$ roots α of $h - 1$ in the unit disc; in particular, they are all in $\{z \in \mathbb{C} \mid a_+ \leq |z| \leq 1\}$. Moreover, there exists an integer $N_2 > N_1$, such that for any $n \geq N_2$,*

$$|g(\alpha)| < n + 2.$$

Proof. Applying Rouché's theorem to $3t^{n+4}$ and $t^{2n+2} - 1$ on the unit circle, we have

$$|3t^{n+4}| = 3 > 2 = |t|^{2n+2} + 1 \geq |t^{2n+2} - 1|.$$

Thus $h(t) - 1 = 3t^{n+4} + t^{2n+2} - 1$ has exactly $n + 4$ roots α in the unit disk. By the maximum modulus principle, for any $|z| < a_+$, $|h(z)| < \max_{|\beta|=a_+} |h(\beta)| \leq h(a_+) = 1$. That is, $h(t) - 1$ has no roots in $\{z \mid |z| < a_+\}$. Hence, the first statement follows.

By the maximum modulus principle for $\tilde{g}$, when $n \geq 2$, we have

$$|g(\alpha)| = |\tilde{g}(\alpha)| \leq \max_{|t|\in\{a_+,1\}} |\tilde{g}(t)| \max\{\tilde{g}(a_+), \tilde{g}(1)\} = \max\{\tilde{g}(a_+), n\}.$$

Noting that $\breve{g}$ is strictly increasing in $\mathbb{R}_{\geq 0}$, we have $\tilde{g}(a_+) = \breve{g}(a_+) < \breve{g}(1 - \frac{1}{n})$ for $n \geq N_1$, by Lemma 2.3. Note $\lim_{n\to\infty} 2(1 - \frac{1}{n})^{(n+1)} = \frac{2}{e}$ and $\lim_{n\to\infty} \frac{(-4+11n-9n^2)}{n^3} = 0$. Take $\epsilon := \frac{1}{2} - \frac{1}{e} > 0$. Then there exists $N_2 > N_1$ such that for any $n \geq N_2$,

$$\begin{aligned}\breve{g}(1 - \frac{1}{n}) &= 2(1 - \frac{1}{n})^{n+1} + \frac{(-4 + 11n - 9n^2)}{n^3} + (n + 1) \\ &< (\frac{2}{e} + \epsilon) + \epsilon + (n + 1) = n + 2.\end{aligned}$$

Hence, $|g(\alpha)| \leq \max\{\tilde{g}(a_+), n\} \leq \max\{\breve{g}(1 - \frac{1}{n}), n\} < n + 2$, whenever $n \geq N_2$. ☐

It remains to discuss the $n-2$ roots of $h-1$ outside the unit disk, which require more careful analysis. The following Lemma 2.6 will be used in the proof of Proposition 2.7.

Lemma 2.6. *For any* $0 < r < \frac{1}{2}$ *and* $n > 10$*, we have*

$$\max_{|z|=r} |(3-z)^3(\frac{n-2}{n} + \frac{2z}{n})^{n-2}| = (3-r)^3(\frac{n-2}{n} + \frac{2r}{n})^{n-2}.$$

Proof. For $|z| = r$, we note $s = \mathrm{Re}(z) \in [-r, r]$ and write

$$|(3-z)^3(\frac{n-2}{n} + \frac{2z}{n})^{n-2}|^2 = \frac{\theta(s)}{n^{2n-4}}, \tag{5}$$

where

$$\theta(s) = (9 - 6s + r^2)^3((n-2)^2 + (4n-8)s + 4r^2)^{n-2}.$$

By direct calculation, we have

$$\begin{aligned}\frac{d\theta}{ds}(s) = {} & 2(9 + r^2 - 6s)^2\big((-2+n)^2 + 4r^2 + 4(-2+n)s\big)^{-3+n} \cdot \\ & \big(2(-2+n)^2(9 + r^2 - 6s) - 9((-2+n)^2 + 4r^2 + 4(-2+n)s)\big) \\ & > 2 \cdot 6^2 \cdot ((n-2)^2 - 2(n-2))^{-3+n} \cdot (2(n-2)^2 \cdot 6 - 9((n-2)^2 + 1 + 2(n-2)) > 0.\end{aligned}$$

Thus (5) is maximized when $s = \mathrm{Re}(z) = r$, i.e., $z = r$. □

Let $r_1 := 0.96$, $r_2 := 1.2$, $\varepsilon_0 := 0.001$, and $i \in \{1, 2\}$. One can check that

$$\varepsilon_0 < \min\{3e^{r_1} - e^{2r_1} - 1, e^{2r_2} - 3e^{r_2} - 1\} \quad \text{and} \quad (3 - \frac{1}{e^{r_i}})^3 e^{\frac{2}{e^{r_i}} + \varepsilon_0} < e^4 - 2\varepsilon_0. \tag{6}$$

These inequalities will be used in the proof of Proposition 2.7.

Proposition 2.7. *There are exactly* $n-2$ *roots* α *of* $h-1$ *in* $\{z \in \mathbb{C} \mid |z| > 1\}$*. Moreover, there exists an integer* $N_3 > 10$*, such that for any* $n \geq N_3$*,*

$$1 + \frac{r_1}{n} \leq |\alpha| \leq 1 + \frac{r_2}{n} \quad \textit{and} \quad |g(\alpha)| < n + 2.$$

Proof. Notice the following two limits

$$\lim_{n\to+\infty} \big(3(1 + \frac{r_1}{n})^{n+4} - (1 + \frac{r_1}{n})^{2n+2} - 1\big) = 3e^{r_1} - e^{2r_1} - 1 > \varepsilon_0,$$
$$\lim_{n\to+\infty} \big((1 + \frac{r_2}{n})^{2n+2} - 3(1 + \frac{r_2}{n})^{n+4} - 1\big) = e^{2r_2} - 3e^{r_2} - 1 > \varepsilon_0,$$

where the inequality follows from (6). Thus there exists $N_3' > 2$, such that for any $n \geq N_3'$, the following (i) and (ii) hold.

(i) For any $|t| = 1 + \frac{r_1}{n}$,

$$|3t^{n+4}| - |t^{2n+2} - 1| \geq 3(1 + \frac{r_1}{n})^{n+4} - (1 + \frac{r_1}{n})^{2n+2} - 1 > \varepsilon_0 > 0.$$

(ii) For any $|t| = 1 + \frac{r_2}{n}$,

$$|t^{2n+2}| - |3t^{n+4} - 1| \geq (1 + \frac{r_2}{n})^{2n+2} - 3(1 + \frac{r_2}{n})^{n+4} - 1 > \varepsilon_0 > 0.$$

By applying Rouché's theorem to (ii), all the $2n+2$ roots β of $h-1$ satisfy $|\beta| \leq 1 + \frac{r_2}{n}$. By applying Rouché's theorem to (i), $h-1$ has exactly $n+4$ roots γ with $|\gamma| \leq 1 + \frac{r_1}{n}$. Then we conclude that $1 + \frac{r_1}{n} \leq |\alpha| \leq 1 + \frac{r_2}{n}$, since $h-1$ already has $n+4$ roots inside the unit disk by Proposition 2.5.

Note $\alpha^{n-2} = \frac{1}{\alpha^{n+4}} - 3$. We have the following (in)equalities.

$$
\begin{aligned}
|\frac{\tilde{g}(\alpha)}{n}|^{n-2} &= \big|(\frac{1}{\alpha^{n+4}}-3)^3(1-\frac{2-\frac{2}{\alpha^{n+4}}}{n})^{n-2}\big| \\
&\le \max_{|t|\in\{1+\frac{r_1}{n},1+\frac{r_2}{n}\}} \big|(\frac{1}{t^{n+4}}-3)^3(1-\frac{2-\frac{2}{t^{n+4}}}{n})^{n-2}\big| \qquad (7)\\
&= \max_{|t|\in\{1+\frac{r_1}{n},1+\frac{r_2}{n}\}} \big|(\frac{1}{|t|^{n+4}}-3)^3(1-\frac{2-\frac{2}{|t|^{n+4}}}{n})^{n-2}\big| \quad (\text{for } n\ge N_4) \qquad (8)\\
&< \max_{i\in\{1,2\}} \big|(\frac{1}{(1+\frac{r_i}{n})^{n+4}}-3)^3(1-\frac{2-\frac{2}{e^{r_i}}-\varepsilon_0}{n})^{n-2}\big| \quad (\text{for } n\ge N_4) \qquad (9)\\
&< \max_{i\in\{1,2\}} (3-\frac{1}{e^{r_i}})^3 e^{\frac{2}{e^{r_i}}+\varepsilon_0-2}+\varepsilon_0 \quad (\text{for } n\ge N_5) \qquad (10)\\
&< e^2-\varepsilon_0 \qquad (11)\\
&< (1+\frac{2}{n})^{n-2} \quad (\text{for } n\ge N_6). \qquad (12)
\end{aligned}
$$

Here the inequality (7) follows from the maximum modulus principle. Since

$$\lim_{n\to\infty}\frac{1}{(1+\frac{r_i}{n})^{n+4}}=\frac{1}{e^{r_i}}<\frac{1}{e^{r_i}}+\frac{\varepsilon_0}{2}<\frac{1}{2},$$

there exists $N_4>10$ such that $\frac{1}{(1+\frac{r_i}{n})^{n+4}}<\frac{1}{e^{r_i}}+\frac{\varepsilon_0}{2}$ holds for any $n\ge N_4$ and $i\in\{1,2\}$. Consequently, the equality (8) holds by Lemma 2.6. The inequalities (9), (10), (12) follow directly from the definition of limit, and the inequality (11) holds by (6).

Hence, we are done, by taking $N_3=\max\{N_3', N_4, N_5, N_6\}$ and noting $g(\alpha)=\tilde{g}(\alpha)$. □

Theorem 2.8. *Let $X=\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(3))$, where $n>\max\{N_2,N_3\}$ and $3\nmid n+1$. Then*

$$\rho<\dim X+1=n+2.$$

Proof. Notice $\rho=\max_{h(t)=1}|g(t)|$, by Proposition 2.1 and the analysis at the beginning of this subsection. Therefore the statement is a direct consequence of the combination of Propositions 2.5 and 2.7. □

Remark 2.9. *When $3\mid n+1$, the curve $x^{n+1}-y^3=0$ is reducible, so the parameterzation $(x,y)=(t^3,t^{n+1})$ is not sufficient. A more involved analysis similar to that for $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(n-1))$ in* [GHIKLS] *may be needed, in order to remove the assumption $3\nmid n+1$.*

Remark 2.10. *Numerical computations by Mathematica 10.0 show that $n=16$ is the smallest number such that Galkin's lower bound conjecture does not hold for $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(3))$, while this conjecture does hold for $\mathbb{P}_{\mathbb{P}^n}(\mathcal{O}\oplus\mathcal{O}(a))$ with $a\in\{2,4,5\}$ and $n<30$.*

In addition to the examples in [GHIKLS], *$\mathbb{P}_{\mathbb{P}^{16}}(\mathcal{O}\oplus\mathcal{O}(3))$ gives a negative answer to Conjecture $\mathcal{O}$ as well.*

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