

```

(* w0E6=
  {2,3,4,5,6,2,3,4,5,6,2,3,4,5,6,2,3,4,5,6,1,3,4,2,5,4,3,1,6,5,4,2,3,4,5,6};
wP1={2,3,4,5,6,2,3,4,5,6,2,3,4,5,6,2,3,4,5,6};
vmat=matweyl[E6,Join[wP1,ww,w0E6]]
  pos=Position[E6mat1[[2,dim[E6,kk]-Length[ww]]],vmat][[1,1]];
vv=E6mat1[[1,dim[E6,kk]-Length[ww],pos]]*)

Print["For  $E_6/P_1$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]

For  $E_6/P_1$ , u={6, 5, 4, 3, 1}
w_0vw_P={1, 3, 4, 5, 2, 4, 3, 1}
v={1, 3, 4, 5, 2, 4, 3, 1}

(*w0E6={2,3,4,5,6,2,3,4,5,6,2,3,4,
  5,6,2,3,4,5,6,1,3,4,2,5,4,3,1,6,5,4,2,3,4,5,6};
wP2={1,3,4,5,6,1,3,4,5,1,3,4,1,3,1}; vmat=matweyl[E6,Join[wP2,ww,w0E6]]
  pos=Position[E6mat2[[2,dim[E6,kk]-Length[ww]]],vmat][[1,1]];
vv=E6mat2[[1,dim[E6,kk]-Length[ww],pos]]*)
Print["For  $E_6/P_2$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]

For  $E_6/P_2$ , u={1, 3, 4, 2}
w_0vw_P={3, 4, 5, 6, 2, 4, 5, 3, 4, 2}
v={6, 5, 4, 3, 2, 4, 5, 1, 3, 4, 2}

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
  6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
wP3 = {1, 2, 4, 5, 6, 2, 4, 5, 2, 4, 2}; vmat = matweyl[E6, Join[wP3, ww, w0E6]];
pos = Position[E6mat3[[2, dim[E6, kk] - Length[ww]]], vmat][[1, 1]];
vv = E6mat3[[1, dim[E6, kk] - Length[ww], pos]];
Print["For  $E_6/P_3$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]

For  $E_6/P_3$ , u={1, 3, 4, 5, 2, 4, 3}
w_0vw_P={4, 3, 2, 4, 5, 6, 4, 3, 2, 4, 5, 4, 3, 2, 4, 1, 3}
v={6, 5, 4, 3, 2, 4, 1, 3}

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
  6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
wP4 = {2, 1, 3, 1, 5, 6, 5}; vmat = matweyl[E6, Join[wP4, ww, w0E6]];
pos = Position[E6mat4[[2, dim[E6, kk] - Length[ww]]], vmat][[1, 1]];
vv = E6mat4[[1, dim[E6, kk] - Length[ww], pos]];
Print["For  $E_6/P_3$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Length[vv]

```

```

For  $E_6/P_3$ ,  $u=\{6, 5, 4, 3, 2, 4\}$ 
w_0vw_P={5, 3, 4, 1, 3, 2, 4, 5, 6, 4, 1, 3, 2, 4, 5, 1, 3, 2, 4}
v={1, 3, 4, 5, 6, 2, 4, 5, 3, 4}
10

```

```

w0F4 = {1, 3, 4, 3, 2, 1, 3, 2, 1, 3, 2, 4, 3, 2, 1, 3, 2, 3, 4, 3, 2, 1, 3, 2};
wP1 = {2, 3, 4, 2, 3, 4, 2, 3, 4};
vmat = matweyl[F4, Join[wP1, ww, w0F4]]
pos = Position[F4mat1[[2, dim[F4, kk] - Length[ww]]], vmat][[1, 1]];
vv = F4mat1[[1, dim[F4, kk] - Length[ww], pos]];
Print["For  $F_4/P_1$ ,  $u=$ ", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Length[vv]
{{-2, -3, -4, -2}, {1, 2, 2, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}

```

```

For  $F_4/P_1$ ,  $u=\{1, 2, 3, 2, 1\}$ 
w_0vw_P={2, 3, 4, 2, 3, 2, 1}
v={1, 2, 3, 4, 2, 3, 2, 1}
8

```

```

w0F4 = {1, 3, 4, 3, 2, 1, 3, 2, 1, 3, 2, 4, 3, 2, 1, 3, 2, 3, 4, 3, 2, 1, 3, 2};
wP2 = {1, 3, 4, 3};
vmat = matweyl[F4, Join[wP2, ww, w0F4]]
pos = Position[F4mat2[[2, dim[F4, kk] - Length[ww]]], vmat][[1, 1]];
vv = F4mat2[[1, dim[F4, kk] - Length[ww], pos]];
Print["For  $F_4/P_2$ ,  $u=$ ", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Length[vv]
{{0, 1, 0, 0}, {-1, -2, -2, -2}, {0, 0, 0, 1}, {1, 2, 3, 1}}

```

```

For  $F_4/P_2$ ,  $u=\{1, 2, 3, 2\}$ 
w_0vw_P={3, 2, 3, 4, 3, 2, 3, 1, 2}
v={1, 2, 3, 4, 2, 3, 1, 2, 3, 1, 2}
11

```

```

w0F4 = {1, 3, 4, 3, 2, 1, 3, 2, 1, 3, 2, 4, 3, 2, 1, 3, 2, 3, 4, 3, 2, 1, 3, 2};
Length[w0F4]
24

```

```

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
        6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP1 = w0E6; vmat = matweyl[E7, Join[wP7, ww, w0E7]];
pos = Position[E7mat7[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat7[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P7, u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For E7/P7, u={7, 6, 5, 4, 3, 2, 4, 5, 6, 7}

w_0vw_P={1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7}

v={7, 6, 5, 4, 3, 2, 4, 5, 6, 7}

length[uu]=10, length[vv]=10

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
        6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {2, 3, 4, 5, 6, 7, 2, 3, 4, 5, 6, 7, 2, 3, 4, 5, 6, 7, 2, 3, 4,
      5, 6, 7, 2, 3, 4, 5, 6, 7}; vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat1[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat1[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P1, u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For E7/P1, u={7, 6, 5, 4, 3, 1}

w_0vw_P={6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3, 1}

v={7, 6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3, 1}

length[uu]=6, length[vv]=17

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
        6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {1, 3, 4, 5, 6, 7, 1, 3, 4, 5, 6, 1, 3, 4, 5, 1, 3, 4, 1, 3, 1};
vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat2[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat2[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P2, u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For E7/P2, u={7, 6, 5, 4, 3, 2, 4, 5, 1, 3, 4, 2}

w_0vw_P=
{5, 6, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 3, 4, 1, 3, 2, 4, 5, 6, 4, 3, 2, 4, 5, 1, 3, 4, 2}

v={7, 6, 5, 4, 3, 2, 4, 5, 1, 3, 4, 2}

length[uu]=12, length[vv]=12

```

```

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5,
        6, 2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 7};
vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat6[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat6[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P6, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]
For E7/P6, u={7, 6, 5, 4, 3, 2, 4, 5, 6}
w_Ovw_P={3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6}
v={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6}
length[uu]=9, length[vv]=15

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5, 6,
        2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {1, 3, 4, 2, 1, 3, 4, 1, 3, 1, 6, 7, 6};
vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat5[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat5[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P5, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]
For E7/P5, u={7, 6, 5, 4, 3, 2, 4, 5}
w_Ovw_P=
{4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 3, 4, 1, 3, 2, 4, 5}
v={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5}
length[uu]=8, length[vv]=18

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5, 6,
        2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {2, 4, 5, 6, 7, 2, 4, 5, 6, 2, 4, 5, 2, 4, 2, 1};
vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat3[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat3[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P3, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]
For E7/P3, u={7, 6, 5, 4, 3, 2, 4, 1, 3}
w_Ovw_P=
{6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 5, 4, 1, 3, 2, 4, 5, 6, 1, 3, 2, 4, 5, 3, 2, 4, 1, 3}
v={7, 6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3}
length[uu]=9, length[vv]=16

```

```

w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5, 6,
        2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
wP = {1, 3, 1, 2, 5, 6, 7, 5, 6, 5};
vmat = matweyl[E7, Join[wP, ww, w0E7]];
pos = Position[E7mat4[[2, dim[E7, kk] - Length[ww]]], vmat][[1, 1]];
vv = E7mat4[[1, dim[E7, kk] - Length[ww], pos]];
Print["For E7/P4, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For E7/P4, u={7, 6, 5, 4, 3, 2, 4, 5, 1, 3, 4}

w_Ovw_P={4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6,
          7, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 3, 4, 1, 3, 2, 4, 5, 3, 4, 1, 3, 2, 4}

v={7, 6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4}

length[uu]=11, length[vv]=15


w0E6 = {2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 2, 3, 4, 5, 6,
        2, 3, 4, 5, 6, 1, 3, 4, 2, 5, 4, 3, 1, 6, 5, 4, 2, 3, 4, 5, 6};
w0E7 = Join[w0E6, E7mat7[[1]][[27]][[1]]];
w0E8P8 = E8mat8[[1]][[57]][[1]];
w0E8 = Join[w0E7, w0E8P8];
wP = w0E7;

vmat = matweyl[E8, Join[wP, ww, w0E8]];
kk = 8; dd = dim[E8, kk];
pos = Position[E8mat8[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat8[[1, dim[E8, kk] - Length[ww], pos]];
Print["For E8/P8, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For E8/P8, u={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8}

w_Ovw_P={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8}

v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8}

length[uu]=12, length[vv]=29


wP = {8, 7, 6, 5, 4, 2, 3, 8, 7, 6, 5, 4, 2, 3, 8, 7, 6, 5, 4,
      2, 3, 8, 7, 6, 5, 4, 2, 3, 8, 7, 6, 5, 4, 2, 3, 8, 7, 6, 5, 4, 2, 3};
vmat = matweyl[E8, Join[wP, ww, w0E8]];
kk = 1; dd = dim[E8, kk];
pos = Position[E8mat1[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat1[[1, dim[E8, kk] - Length[ww], pos]];
Print["For E8/P1, u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

```

```

For  $E_8/P_1$ ,  $u=\{8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3, 1\}$ 

w_Ovw_P={2, 4, 5, 6, 7, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8,
  2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3, 1}

v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5, 2, 4, 3, 1}

length[uu]=18, length[vv]=29


wP = {1, 3, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 1, 3, 4, 5, 6, 1, 3, 4, 5, 1, 3, 4, 1, 3, 1};
vmat = matweyl[E8, Join[wP, ww, w0E8]];
kk = 2; dd = dim[E8, kk];
pos = Position[E8mat2[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat2[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_2$ , u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For  $E_8/P_2$ ,  $u=\{8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 2\}$ 

w_Ovw_P=
{5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8, 3, 4, 5, 6, 2, 4,
  5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 5, 4, 3, 2, 4, 5, 1, 3, 4, 2}

v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5, 1, 3, 4, 2}

length[uu]=22, length[vv]=29


wP = Join[w0E6, {8}];
vmat = matweyl[E8, Join[wP, ww, w0E8]];
kk = 7; dd = dim[E8, kk];
pos = Position[E8mat7[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat7[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_7$ , u=", Reverse[uu]];
Print["w_Ovw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]

For  $E_8/P_7$ ,  $u=\{8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7\}$ 

w_Ovw_P={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7,
  8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7}

v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7}

length[uu]=18, length[vv]=28

```

```

wOE8P8 = E8mat8[[1]][[57]][[1]];
wP = {1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 1, 3, 4, 2, 5, 7, 8, 7};
wOE8 = Join[wOE7, wOE8P8];
vmat = matweyl[E8, Join[wP, ww, wOE8]];
kk = 6; dd = dim[E8, kk]; len = 73;
pos = Position[E8mat6[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat6[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_6$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]
For  $E_8/P_6$ , u={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6}
w_0vw_P={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5,
3, 4, 1, 3, 2, 4, 5, 6, 7, 8, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4,
5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6}
v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6}
length[uu]=22, length[vv]=27

wP = {1, 2, 4, 5, 6, 7, 8, 2, 4, 5, 6, 7, 2, 4, 5, 6, 2, 4, 5, 2, 4, 2};
vmat = matweyl[E8, Join[wP, ww, wOE8]];
kk = 3; dd = dim[E8, kk]; len = 73;
pos = Position[E8mat3[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = E8mat3[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_6$ , u=", Reverse[uu]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[uu], ", length[vv]=", Length[vv]]
For  $E_8/P_6$ , u={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3}
w_0vw_P={2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8, 5, 4, 3, 2, 4,
5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 8, 3, 4, 5, 6, 2,
4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 7, 6, 5, 4, 3, 2, 4, 5, 6, 1, 3, 4, 5, 2, 4, 3}
v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5, 2, 4, 3}
length[uu]=23, length[vv]=28

wP = {1, 3, 4, 2, 1, 3, 4, 1, 3, 1, 6, 7, 8, 6, 7, 6};
vmat = matweyl[E8, Join[wP, ww, wOE8]];
kk = 5; dd = dim[E8, kk]; len = 73;
pos = Position[mat5bP5[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = mat5bP5[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_5$ , u=", Reverse[u1]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[u1], ", length[vv]=", Length[vv]]
For  $E_8/P_5$ , u={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5}
w_0vw_P={7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4,
5, 6, 7, 5, 6, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 3, 4, 1, 3, 2, 4, 5, 4, 3, 2, 4, 1,
3, 8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5}
v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5}
length[uu]=25, length[vv]=26

```

```

wP = {1, 3, 1, 2, 5, 6, 7, 8, 5, 6, 7, 5, 6, 5};
vmat = matweyl[E8, Join[wP, ww, w0E8]];
kk = 4; dd = dim[E8, kk]; len = 73;
pos = Position[mat4bP4[[2, dim[E8, kk] - Length[ww]]], vmat][[1, 1]];
vv = mat4bP4[[1, dim[E8, kk] - Length[ww], pos]];
Print["For  $E_8/P_4$ , u=", Reverse[u1]];
Print["w_0vw_P=", Reverse[ww]];
Print["v=", Reverse[vv]]
Print["length[uu]=", Length[u1], ", length[vv]=", Length[vv]]
For  $E_8/P_4$ , u={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4}

w_0vw_P={6, 5, 4, 3, 2, 4, 5, 6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5,
  6, 7, 1, 3, 4, 5, 6, 2, 4, 5, 3, 4, 1, 3, 2, 4, 5, 6, 5, 4, 3, 2, 4, 5, 1, 3, 4,
  2, 8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5, 2, 4}

v={8, 7, 6, 5, 4, 3, 2, 4, 5, 6, 7, 8, 1, 3, 4, 5, 6, 7, 2, 4, 5, 6, 3, 4, 5, 2, 4}

length[uu]=25, length[vv]=27

```

In[221]:=

```

len = 8;
mat = hasse[F4, 1, len];
lenmat = Length[mat];
g = FromAdjacencyMatrix[mat, Type → Directed];
h = HasseDiagram[SetVertexLabels[g,
  Join[{" $\sigma_1$ "}, Table["", {i, 1, 4}], {Style[" $\sigma_5$ ", Red]}, Table["", {i, 7, 10}],
    {Style[" $\sigma_7$ ", Red]}, Table["", {i, 9, lenmat - 4}], { $\sigma_{len}$ }}]]];
Labeled[ShowGraph[h, BaseStyle → {FontSize → 18}], "F4(1)"];

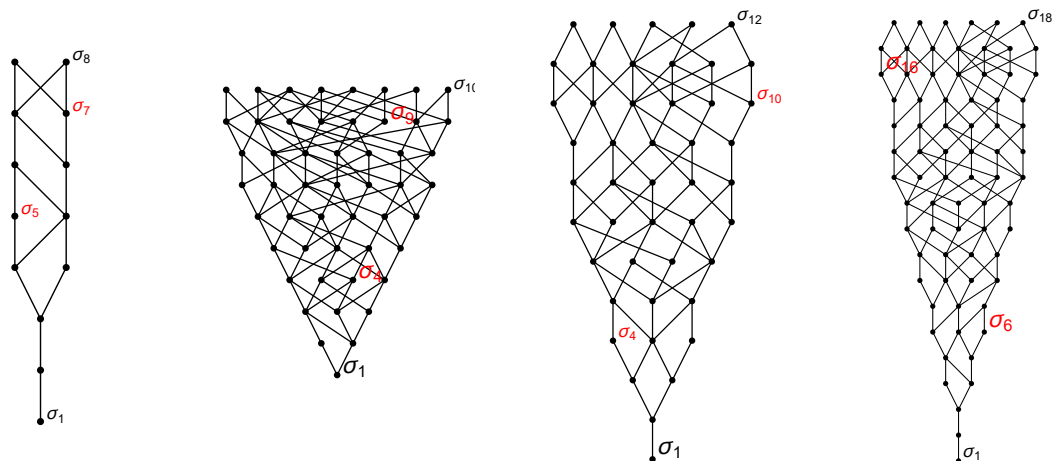
len = 12;
matE62 = hasse[E6, 2, len];
lenmat = Length[matE62];
g62 = FromAdjacencyMatrix[matE62, Type → Directed];
h62 = HasseDiagram[SetVertexLabels[g62,
  Join[{" $\sigma_1$ "}, {"", "", "", Style[" $\sigma_4$ ", Red, Small]}, Table["", {i, 5, lenmat - 13}],
    {Style[" $\sigma_{10}$ ", Red, Small]}, Table["", {i, lenmat - 11, lenmat - 2}], { $\sigma_{len}$ }}]]];
ShowGraph[h62, BaseStyle → {FontSize → 18}];

len = 18;
matE71 = hasse[E7, 1, len];
lenmat = Length[matE71];
g71 = FromAdjacencyMatrix[matE71, Type → Directed];
h71 = HasseDiagram[
  SetVertexLabels[g71, Join[{" $\sigma_1$ "}, Table["", {i, 1, 8}], {Style[" $\sigma_6$ ", Red]},
    Table["", {i, 10, lenmat - 2 - 19}], {Style[" $\sigma_{16}$ ", Red, Tiny]},
    Table["", {i, lenmat - 2 - 19 + 2, lenmat - 2}], { $\sigma_{len}$ }}]]];
ShowGraph[h71, BaseStyle → {FontSize → 18}];

len = 10;
mat = hasse[F4, 2, len];
lenmat = Length[mat];
g42 = FromAdjacencyMatrix[mat, Type → Directed];
h42 = HasseDiagram[SetVertexLabels[g42, Join[{" $\sigma_1$ "}, Table["", {i, 1, 7}],
  {Style[" $\sigma_4$ ", Red]}, Table["", {i, 10, lenmat - 11}], {Style[" $\sigma_9$ ", Red]},
  Table["", {i, lenmat - 9, lenmat - 2}], {"", Style[ $\sigma_{len}$ , Small]}]]];
ShowGraph[h42, BaseStyle → {FontSize → 18}];
ShowGraphArray[{h, h62, h71}];
ShowGraphArray[{h, h42, h62, h71}]

```

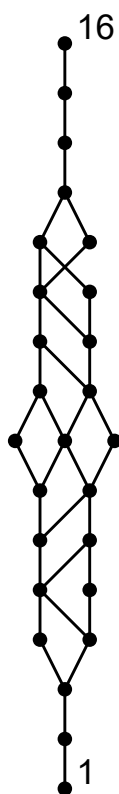
Out[116]=



```

len = 16;
mat = hasse[E6, 1, len];
lenmat = Length[mat];
g = FromAdjacencyMatrix[mat, Type -> Directed];
h = HasseDiagram[
  SetVertexLabels[g, Join[{1}, Table["", {i, 1, lenmat - 2}], {len}]]];
Labeled[ShowGraph[h, BaseStyle -> {FontSize -> 18}], "E6/P1"]

```

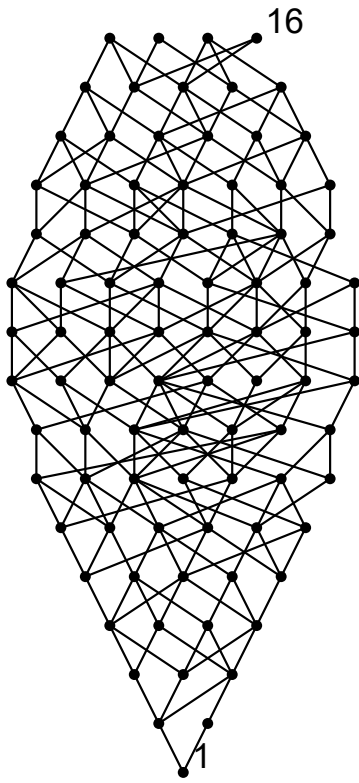


E6/P1

```

len = 16;
mat = hasse[F4, 3, len];
lenmat = Length[mat];
g = FromAdjacencyMatrix[mat, Type → Directed];
h = HasseDiagram[
  SetVertexLabels[g, Join[{1}, Table["", {i, 1, lenmat - 2}], {len}]]];
Labeled[ShowGraph[h, BaseStyle → {FontSize → 18}], "F4/P3"]

```



F4/P3